

Temperature and Disorder Chaos in Low Dimensional Directed Paths

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The responses of a $1 + \varepsilon$ dimensional directed path to temperature and to potential variations are calculated exactly and are governed by the same scaling form. The short scale decorrelation (strong correlation regime) leads to the overlap length predicted by heuristic approaches; its temperature dependence and large absolute value agree with scaling and numerical observations. Beyond the overlap length (weak correlation regime), the correlation decays algebraically. A clear physical mechanism explains the behavior in each case: the initial decorrelation is due to “fragile droplets,” which contribute to the entropy fluctuations as \sqrt{T} , while the residual correlation results from accidental intersections of otherwise uncorrelated configurations.

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Many disordered systems are said to be “complex” because they operate in the vicinity of a large number of microscopically distinct, metastable states. This generic property has several nontrivial manifestations, eminently, slow dynamics and aging, irreversibility, and the relevance of preparation history. A related aspect is the *fragility*, or *extreme sensitivity*, of disordered systems: the “dominant” metastable states shift from given microscopic configurations to completely different ones upon even a slight variation of (external) parameters, a behavior that has been termed “chaotic” as small perturbations map into major reconfigurations [1–4].

In experiments, temperature plays the role of a prototypical control parameter as it is far easier to vary than, say, the microscopic details of the disorder. *Temperature chaos* (TC) stipulates that, rather remarkably, the configurations probed by a disordered system (in its glassy phase) at temperature T are largely *uncorrelated* to the ones probed at a slightly different temperature $T + \delta T$, provided the system is large enough, larger in fact than a characteristic *overlap length* L^* (which for consistency must diverge as $\delta T \rightarrow 0$) [2–4]. This scenario may provide a natural explanation for several of the peculiar dynamical properties of spin glasses, such as the absence of cooling rate effects and the “rejuvenation” phenomenon [5,6]. Indeed, if a sufficiently large temperature step modifies drastically the equilibrium configurations, it is equivalent in its effect to a quench from high temperatures, regardless of the dwelling time at the high temperatures. Numerical observations of dynamical rejuvenation in the absence of any direct evidence of static TC [7,8], however, seem to point to alternative pictures which do not rely on an overlap length but rather on the gradual freezing at successive scales [8–10]. While these are compatible with the large value (in absolute terms) of

L^* inferred from numerics [4], as well as from mean-field [11] and Migdal-Kadanoff [12] approximations, which would render the overlap length irrelevant in the face of the small dynamical length scales in spin glasses (a few tens of lattice spacings even on experimental time scales), recent arguments suggest that TC may manifest itself in ways relevant to experiments even at scales much smaller than L^* [5,13].

From a theoretical point of view, systematic investigations of TC are scarce. In the context of spin glasses, calculations have been confined to mean-field [11] and Migdal-Kadanoff [12,14] approximations, begging for studies of simpler models that still contain the physics of TC. Unfortunately, the exactly soluble random energy model does not display TC (except in the vicinity of the critical temperature [15]), unless it is extended to include explicit random entropy variables [16]. The next best candidate for a theoretical treatment is directed paths (polymers) in a random potential. The sensitivity of an array of pinned flux lines to potential and temperature variations was studied by renormalization group methods [17], which yielded scaling forms (identical in both cases) in the strong correlation regime. However, neither the dependence of the overlap length on temperature nor the weak correlation regime were elucidated. In the case of a single directed path, the (somewhat nonintuitive) result that changing temperature is “equivalent” to changing disorder is confirmed by extensive numerics (in Ref. [18], which also advances arguments based on the mapping to bosons [19]). The present work characterizes TC and disorder chaos (DC) for a directed path *exactly*, by working close to $d = 1$ dimension, where a systematic $\varepsilon = d - 1$ expansion can be performed [20,21]. To our knowledge, this represents the first systematic, complete derivation of these effects; our exact results broadly confirm

and complement the predictions of scaling arguments at strong correlation, elucidate the behavior at weak correlation, and match a clear picture of the physical mechanisms involved in each regime.

Let us first summarize, in the context of directed paths, the scaling arguments for TC and DC. The crucial assumption is that the free energy of a directed path in a random potential consists, in the low temperature phase, of the sum of a nonrandom term growing linearly with the length L of the path, with a prefactor regular in T , and, almost surely, a random contribution ΔF of order L^θ with $\theta < 1/2$. (For example, $\theta = 1/3$ holds exactly for two dimensional paths.) Now, suppose that the random potential is slightly modified by a small unbiased random addition of order δV per bond, and assume $T = 0$ for the sake of simplicity. The resulting extra contribution to the energy of the unperturbed ground state is obviously a random variable of order $\delta VL^{1/2}$, which, for large L , exceeds the postulated L^θ fluctuations. To avoid a contradiction, a new ground state configuration, roughly uncorrelated to the unperturbed one, must appear when $\delta VL^{1/2} > L^\theta$, i.e., when $L > L^* \sim (\delta V)^{-2/(1-2\theta)}$, the overlap length corresponding to *potential variations*. The case of *temperature variations* is more subtle. At $T > 0$, configurations that do not optimize the energy contribute normal *energy* fluctuations ΔE , of order $L^{1/2}$, and similarly for *entropy* fluctuations ΔS , such that the two $L^{1/2}$ contributions precisely cancel and give way to the expected L^θ *free energy* fluctuations ΔF . Now, if one could use the thermodynamical identity $\partial\Delta F/\partial T = -\Delta S$, one would conclude that the fluctuations of the free energy vary, upon a change of temperature, by an amount $\Delta S\delta T \sim L^{1/2}\delta T$, which again exceeds the “allowed” L^θ fluctuations whenever $L > L^* \sim (\delta T)^{-2/(1-2\theta)}$. The only way out is that the “dominant” configurations shift to radically different ones, inducing nonanalyticities in ΔF that can be seen as a sequence of microphase transitions at all temperatures. That, in the glassy phase, disordered systems are in a sense critical at all temperatures, and therefore extremely sensitive to parameter changes, was suggested in the context of mean-field spin glasses [22]. This generic fragility was predicted by Fisher and Huse [2,4], as part of a rich scaling picture from which the above arguments are extracted (see also Refs. [3,17,23]).

We now turn to our analytical results and their physical interpretation. (A full account of the technical details will be published separately.) We study the statistical mechanics of directed paths on a standard Berker lattice constructed recursively by replacing a bond by a diamond with b two-bond branches. Each bond carries a Gaussian random energy with vanishing mean and standard deviation σ_1 (though our results presumably hold more generally, at least for sufficiently short tailed densities). To uncover the effect of a small change in either temperature or random potential, we focus on two paths at temperatures T and T' , and subjected to two random potentials

with a correlation coefficient ρ_1 , and we ask how they decorrelate as a function of their length L . Our calculation generalizes the beautiful work of Derrida and Griffiths [20] to nonvanishing temperatures [21] and to include more than one directed path. We establish an exact recursion relation for the joint probability distribution $\mathcal{P}(F(T), F'(T'))$ of the free energies of the two directed paths. This recursion relation can be solved in the low dimensional limit $b = 1 + \varepsilon$, in which \mathcal{P} is close to a bivariate Gaussian distribution. The averages and higher (joint) moments of the free energies may be extracted from this distribution; in line with our central question, we focus on the normalized correlation

$$\rho = \frac{\overline{\delta F(T)\delta F'(T')}}{\sqrt{\overline{\delta F(T)^2}\overline{\delta F'(T')^2}}}, \quad (1)$$

with $\delta F^{(i)}(T^{(i)}) = F^{(i)}(T^{(i)}) - \overline{F^{(i)}(T^{(i)})}$. The quantity ρ codifies the sensitivity of a directed path to temperature or random potential variations

The free energies of a pair of shortest possible paths are the bond energies themselves, and thus $\rho(L=1) \equiv \rho_1$; $\rho_1 = 1$ if the two random potentials felt by the directed paths are identical, while $0 < \rho_1 < 1$ if they are different (but correlated). For longer and longer paths, ρ falls to zero due to thermal fluctuations (if $T \neq T'$) or to quenched fluctuations (if the random potentials are different). Our calculation recovers the scaling form of the initial decorrelation in the *strong correlation regime* and yields the overlap length L^* (appearing here as the scale at which ρ has fallen by a fraction, say 1/2, of its initial value) in terms of the perturbation. In the *weak correlation regime*, beyond L^* , we find that the residual correlation ρ decays much more slowly than was previously anticipated.

At short scales or strong correlation, the initial decorrelation evolves, as a function L , according to

$$\rho = 1 - \Delta L^{1-2\theta} + O(L^{-1/2-2\theta}), \quad (2)$$

where, in the limit $1 - \rho_1 \ll 1$ and $|\delta T| \ll \sigma_1$,

$$\Delta = (1 - \rho_1) + \mathcal{A}s(T)\left(\frac{\delta T}{\sigma_1}\right)^2. \quad (3)$$

In these expressions,

$$\theta = \frac{1}{2} - \frac{K_2}{\ln(2)}\varepsilon \quad (4)$$

is the exponent ruling free energy fluctuations [20,21],

$$s(T) = \frac{\pi^2}{6(2\sqrt{2}-1)}K_1\frac{T}{\sigma_1} \quad (5)$$

is the low temperature entropy per unit length, and the numerical constants \mathcal{A} , K_1 , and K_2 take the values

$$\mathcal{A} = \frac{3}{\pi^2} \int_0^1 du \left[\frac{u \ln^2(u)}{(u+1)^2} + 2 \frac{\ln^2(u+1)}{u} \right] \approx 0.23, \quad (6)$$

$$K_n = - \int_{-\infty}^{\infty} du u^{n-1} E(u) \ln(E(u)). \quad (7)$$

[$E(u) = \int_{-\infty}^u dx / \sqrt{2\pi} e^{-x^2/2}$ denotes the usual error function.] While our formulation is valid at any temperature, the above explicit expressions correspond to the dominant low temperature behavior. More generally, we find that Δ is a regular function of $(\delta T)^2$, at odds with “weak chaos” arguments [5,18] which naively suggest an additional, singular $|\delta T|^3$ contribution.

The above result for the initial decorrelation of a directed path in a random potential may be viewed as follows. From the calculation, it appears clearly that Δ embodies the “ignition” of the decorrelation, which occurs dominantly at short scales. A path of length L contains a number $\propto L$ of fragile “ignition droplets” responsible for the decorrelation at short scales; their contribution to the overall decorrelation is therefore proportional to L divided by the square of the free energy fluctuations, i.e., to $L^{1-2\theta}$, as in Eq. (2) and in agreement with recent numerical data [18] on directed paths in $d = 1 + 1$ dimensions. Hence, Δ can be thought of as the density of ignition droplets. (Equivalently, Δ corresponds to the probability that an arbitrary small droplet takes part in the “ignition.” Pushing this line of reasoning further, one can infer from the calculation that a droplet of size ℓ takes part in the ignition with probability $\propto 1/\ell^\theta$, consistent with the dominance of small droplets and the predictions of scaling theory [4].) This interpretation becomes transparent in the case of TC if we consider, for the sake of simplicity, $T' = 0$ and $T = \delta T$ small. Then the correlation reads

$$\rho \approx 1 - \frac{1}{2} \left\{ \frac{\overline{T^2(S - \bar{S})^2}}{(\overline{E_0 - \bar{E}_0})^2} - \left[\frac{\overline{(E_0 - \bar{E}_0)T(S - \bar{S})}}{(\overline{E_0 - \bar{E}_0})^2} \right]^2 \right\}, \quad (8)$$

where E_0 is the ground state energy of the path and $S = s(T)L$ its entropy. Now, the entropy of the path at temperature T comes from a number of droplets, distinguishing nearly degenerate paths, such that the energy difference between the arms of the droplet differ by less than T . These droplets have a certain density per unit length, proportional to $s(T)$ [roughly, $s(T)/\ln(2)$]. If we compare two samples, corresponding to two different realizations of the random potential, some of the droplets existing (at given positions on the path) in the first sample will not be present in the second and vice versa, so that the *variance* of the entropy too grows linearly with the length of the path. This picture of Poissonian droplets implies furthermore that the *variance of the entropy* is given by the *entropy itself*, i.e., we expect $\overline{(S - \bar{S})^2} \propto s(T)L$. With $\overline{(E_0 - \bar{E}_0)(S - \bar{S})} \ll \overline{(S - \bar{S})^2}$ for large L and the scaling of ground state energy fluctuations $\overline{(E_0 - \bar{E}_0)^2} \sim \sigma_1^2 L^{2\theta}$, Eq. (8) then reduces to

$$\rho \approx 1 - \frac{1}{2} s(T) \cdot \left(\frac{\delta T}{\sigma_1} \right)^2 \cdot L^{1-2\theta}, \quad (9)$$

in agreement with Eqs. (2) and (3). A similar argument may be constructed in the case of DC.

A notable outcome of this discussion, substantiated by the analytic result, is the fact that sample to sample variations of the entropy are of order \sqrt{T} —a behavior that extends to spin glasses on Berker lattices [12]. As mentioned, this is due to the *intermittent* nature of the droplets contributing to entropy: a droplet is active with probability $\propto T$, and inactive otherwise. Therefore all moments of the entropy, including the mean and the variance, are proportional to T .

To conclude the discussion of the strong correlation regime, we note that, according to Eq. (2), once $L^{1-2\theta}$ becomes of order $1/\Delta$ the correlation has dropped by a significant fraction of its initial ($L = 1$) value. For TC, this observation defines an overlap length

$$L^* \approx \left(\sqrt{\mathcal{A} s(T)} \frac{\delta T}{\sigma_1} \right)^{-1/(1/2-\theta)} \sim (\sqrt{T} \delta T)^{-1/(1/2-\theta)}, \quad (10)$$

in agreement with the outcome of the Fisher-Huse scaling theory [4]. Inserting numerical values, we find a rather large overlap length, $L^* \approx 10^4$, even for $T = \sigma_1$, $\delta T = \sigma_1/2$, and $\theta = 1/3$ (the exact value in $d = 1 + 1$ dimensions), in qualitative agreement with numerical observations for both directed paths [4,18] and spin glasses [12]. Finally, since the (small) fragile droplets dominate the ignition mechanism for TC, consistency requires that L^* be much larger than the typical distance $\ell_T \propto 1/T$ between droplets. From Eq. (10), this is indeed the case as long as $0 < \theta < 1/2$.

In the weak correlation regime with $L \gg L^*$, we find, at odds with common expectations, a slow algebraic decay

$$\rho \sim L^{-(1-2K_2)\varepsilon/\ln(2)} \quad (11)$$

for both temperature and disorder decorrelation. Again, a simple physical picture explains this behavior. As confirmed by our calculation, at large L the ground state dominates the statistics or, equivalently, the temperature effectively scales to zero. Thus, whether at slightly different temperatures or in slightly different random potentials, our two directed paths behave at large scales as if *pinned* by two *uncorrelated* random energy configurations, and their residual correlation arises from the (rare) bonds at which they intersect. Whence we expect

$$\rho = \frac{\overline{(E_0 - \bar{E}_0)(E'_0 - \bar{E}_0)}}{(\overline{E_0 - \bar{E}_0})^2} \approx \frac{I(L)(e_I - \bar{e}_I)^2}{(\overline{E_0 - \bar{E}_0})^2}, \quad (12)$$

where $I(L)$ is the average number of intersection bonds and e_I is the energy of an intersection bond. Now, for completely uncorrelated paths on a Berker lattice with branching b , $I(L) = L^{1-\ln(b)/\ln(2)}$ trivially. Extrapolating this result to $b = 1 + \varepsilon$, we note that the quantity

$$\frac{I(L)}{(\overline{E_0 - \bar{E}_0})^2} \sim \frac{L^{1-\ln(b)/\ln(2)}}{L^{2\theta}} \sim L^{-(1-2K_2)\varepsilon/\ln(2)} \quad (13)$$

precisely coincides with the analytic result of Eq. (11). From this argument, we conclude that the intersections occur indeed “accidentally”: free energy minimization manifests itself, at best, subdominantly.

The physical mechanism for residual correlation just described does not bear on any specificity of the model and affords us with natural conjectures for the large L decorrelation of directed paths in higher dimensions. For two paths in $d = 1 + D$ dimensions, emerging from the same end point but otherwise uncorrelated (on scales $L \gg L^*$), the probability of encounter at a distance ℓ away from the end point is $1/\ell^{\zeta D}$, where $\zeta = (1 + \theta)/2$ is the wandering exponent. Therefore, the mean number of accidental intersections, at large L , grows like $L^{1-\zeta D}$ if $\zeta D < 1$ and converges to a constant if $\zeta D > 1$, so that we expect the correlation to scale as $\rho \sim L^{1-\zeta D-2\theta}$ if $\zeta D < 1$ and $L^{-2\theta}$ if $\zeta D > 1$. For example, in $d = 1 + 1$ dimensions, $\zeta = 2/3$ and $\theta = 1/3$, whence $\rho \sim L^{-1/3}$ for $L \gg L^*$ (instead of the previously conjectured $L^{-4/3}$ decay [4]). It is unfortunately impossible to check this prediction against recent numerical results [18], as these do not extend to the asymptotic decorrelation regime.

In sum, while the decorrelation mechanisms—entropic fluctuations of fragile droplets for TC and energetic competition for DC—are different, the scaling form of the free energy correlation, when expressed in terms of L/L^* , is exactly the same in the two cases. We calculated exactly this scaling form and the dependence of the overlap length L^* on either temperature or potential variations. Perhaps surprisingly, the free energy correlation decays as a weak power law, rather than exponentially, at large L . A study of the statistics of the overlap between unperturbed and perturbed paths, possibly using similar methods, would be an interesting extension of the present work to better understand the crossover from short to long scales and the nature of the overlap length. In another worthwhile extension, one could apply the systematic approach on low dimensional Berker lattices to study TC in spin glasses. However, beyond its added calculational complication, the case of spin glasses might be more subtle due to the presence of several exponents ruling the glassy phase [24], instead of a single one, θ , for directed paths. Physically, the presence of several exponents reflects the fact that the scaling of energy fluctuations depends on the topological nature of excitations (that may be compact, spongelike, or fractal), and it might well be that on Euclidean lattices a backbone of “strong links” prevents a complete decorrelation for small temperature changes. An interesting variation on the directed path problem, that may mimic this mechanism, incorporates a fat tailed random potential V [distributed according to a power law $P(V)$], as in that case the directed path configurations are controlled by the particularly favorable sites. In $d = 1 + 1$ dimensions, a

simple argument suggests that TC exists only if $P(V)$ decays slower than $V^{-9/2}$. From a theoretical point of view, a finer understanding of such a behavior is, doubtless, worthwhile.

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